Published by Institute of Physics Publishing for SISSA/ISAS
Received: *August 30, 2004*
Revised: *December 2, 2004*
Accepted: *December 9, 2004*

Finite $\mathrm{SU}(N)^k$ unification

Ernest Ma*
Physics Department, University of California
Riverside, California 92521, U.S.A.
E-mail: ma@phyun8.ucr.edu

Myriam Mondragón†
Inst. de Fisica, Univ. Nacional Autonoma de Mexico
Mexico 01000, D.F., Mexico
E-mail: myriam@fisica.unam.mx

George Zoupanos‡
Physics Department, National Technical University of Athens
Athens, Greece
E-mail: george.zoupanos@cern.ch

Abstract: We consider $N = 1$ supersymmetric gauge theories based on the group $\mathrm{SU}(N)_1 \times \mathrm{SU}(N)_2 \times \cdots \times \mathrm{SU}(N)_k$ with matter content $(N, N^*, 1, \ldots, 1) + (1, N, N^*, \ldots, 1) + \cdots + (N^*, 1, 1, \ldots, N)$ as candidates for the unification symmetry of all particles. In particular we examine to which extent such theories can become finite and we find that a necessary condition is that there should be exactly three families. We discuss further some phenomenological issues related to the cases $(N, k) = (3, 3)$, $(3, 4)$, and $(4, 3)$, in an attempt to choose those theories that can become also realistic. Thus we are naturally led to consider the $\mathrm{SU}(3)^3$ model which we first promote to an all-loop finite theory and then we study its additional predictions concerning the top quark mass, Higgs mass and supersymmetric spectrum.

Keywords: Supersymmetry Phenomenology, Beyond Standard Model, Supersymmetric Standard Model, GUT.

*Supported in part by the U.S. Department of Energy under Grant No. DE-FG03-94ER40837.
†Supported in part by the mexican grants PAPIIT-UNAM IN116202, and Conacyt grant 42026-F.
‡Partially supported by the programmes of Ministry of Education "ΠΥΘΑΓΟΡΑΣ" and "ΗΡΑΚ ΛΕΙΤΟΣ" and by the NTUA programme "ΘΑΛΗΣ".

JHEP12(2004)026

Contents

1. Introduction

In the last years there has been a large and sustained effort from the theoretical particle physics community to produce a unified description of all interactions. Two main frameworks have emerged during this endeavor, namely the superstring and the non-commutative geometry. The two approaches, although at a different stage of development, have common unification targets and share similar hopes for exhibiting improved renormalization properties in the ultraviolet (UV) regime as compared to ordinary field theories. Moreover, the two frameworks came closer by the observation that a natural realization of non-commutativity of space appears in the string theory context of D-branes in the presence of a constant background antisymmetric field [1, 2]. However, despite the importance of having frameworks to discuss quantum gravity in a self-consistent way and possibly to construct there finite theories, it is very interesting to search for the minimal realistic framework where finiteness can appear; the history of physics taught us that new ideas might work but usually this happens in the minimal setting. Furthermore, it is interesting to note that non-commutative gauge theories instead of being finite exhibit a curious mixing between the short and long distance modes in their loop expansion, called UV/IR mixing. For a theory to be finite in this framework it has to be finite beforehand in the continuum [3]. The aim for finiteness fulfilling an old theoretical dream remained central in various theoretical efforts over decades even in unrealistic frameworks from the particle physics point of view, such as the supersymmetric N=4 gauge theories starting in '80s [4], up to the AdS/CFT correspondence [5, 6, 7] observed in $AdS_5 \times S_5$ compactification of type IIB superstrings.

In a different context, the main goal expected from a unified description of interactions by the particle physics community is to understand the present day large number of free parameters of the Standard Model (SM) in terms of a few fundamental ones. In other words: to achieve *reduction of couplings* at a more fundamental level.

In our recent studies [8]–[10] we have developed a complementary strategy in searching for a more fundamental theory possibly at the Planck scale, whose basic ingredients are GUTs and supersymmetry, but its consequences certainly go beyond the known ones. Our method consists of hunting for renormalization group invariant (RGI) relations holding below the Planck scale, possibly set or required in a more fundamental theory, which in turn are preserved down to the GUT scale. This programme, called Gauge-Yukawa unification scheme, applied in the dimensionless couplings of supersymmetric GUTs, such as gauge and Yukawa couplings, had already noticable successes by predicting correctly, among others, the top quark mass in the finite SU(5) GUTs. An impressive aspect of the RGI relations is that one can guarantee their validity to all-orders in perturbation theory by studying the uniqueness of the resulting relations at one-loop, as was proven in the early days of the programme of *reduction of couplings* [11]. Even more remarkable is the fact that it is possible to find RGI relations among couplings that guarantee finiteness to all-orders in perturbation theory [12]–[14], including the soft supersymmetry breaking sector.

Here we examine the construction of realistic FUTs based on product gauge groups. In particular we point out that finiteness actually determines the number of families n_f in a class of supersymmetric $\mathrm{SU}(N)^k$ gauge theories, namely $n_f = 3$ regardless of N and k. The case $N = 4$ and $k = 3$ was first pointed in ref. [15], and that of arbitrary N and $k = 3$ was discussed in ref. [16], both from the string point of view. Concerning the soft supersymmetry breaking sector of these latter models, although in principle it could be understood too in the same framework under certain assumptions [15, 17, 18], the explicit construction is still missing.

Our search for realistic FUTs based on product groups leads us to choose a supersymmetric $\mathrm{SU}(3)^3$ model, which we subsequently promote to an all-loop finite theory, whose predictions we examine further.

The rest of the paper is organised as follows: in section 2 we review the method of reduction of couplings and recall how it is applied in $N = 1$ supersymmetric gauge theories in order to obtain all-loop finite gauge theories. In section 3 we describe the extension of finiteness in the case of soft supersymmetry breaking terms. Section 4 is devoted to a search for realistic FUTs based on product groups, out of which an $\mathrm{SU}(3)^3$ supersymmetric gauge theory with three families is singled out. This theory then is further discussed in detail in section 5. Section 6 contains the predictions of the $\mathrm{SU}(3)^3$ FUT concerning the top quark mass, the Higgs boson masses and the supersymmetric spectrum.

2. Reduction of couplings and finiteness in $N = 1$ supersymmetric gauge theories

Let us first recall the basic issues concerning *reduction of couplings*, in the case of dimensionless couplings and *finiteness* of $N = 1$ supersymmtric theories.

A RGI relation among couplings g_i,

$$\mathcal{F}(g_1, \ldots, g_N) = 0\,, \tag{2.1}$$

has to satisfy the partial differential equation

$$\mu \, \frac{d\mathcal{F}}{d\mu} = \sum_{i=1}^{N} \beta_i \, \frac{\partial \mathcal{F}}{\partial g_i} = 0 \,, \tag{2.2}$$

where β_i is the β-function of g_i. There exist $(N-1)$ independent $\mathcal{F}$'s, and finding the complete set of these solutions is equivalent to solve the so-called reduction equations (REs) [11],

$$\beta_g \left(\frac{dg_i}{dg} \right) = \beta_i \ , \ i = 1, \ldots, N \,, \tag{2.3}$$

where g and β_g are the primary coupling and its β-function. Using all the $(N-1)$ $\mathcal{F}$'s to impose RGI relations, one can in principle express all the couplings in terms of a single coupling g. The complete reduction, which formally preserves perturbative renormalizability, can be achieved by demanding a power series solution, whose uniqueness can be investigated at the one-loop level.

In order to discuss finiteness, it seems unavoidable that we should consider supersymmetric gauge theories. Let us then consider a chiral, anomaly free, $N = 1$ globally supersymmetric gauge theory based on a group G with gauge coupling constant g. The superpotential of the theory is given by

$$W = \frac{1}{2} m^{ij} \, \Phi_i \, \Phi_j + \frac{1}{6} C^{ijk} \, \Phi_i \, \Phi_j \, \Phi_k \,, \tag{2.4}$$

where m^{ij} and C^{ijk} are gauge invariant tensors and the matter field Φ_i transforms according to the irreducible representation R_i of the gauge group G. All the one-loop β-functions of the theory vanish if $\beta_g^{(1)}$ and all the anomalous dimensions of the superfields $\gamma_i^{j(1)}$ vanish, i.e.

$$\sum_i \ell(R_i) = 3 C_2(G) \,, \qquad \frac{1}{2} C_{ipq} C^{jpq} = 2 \delta_i^j g^2 C_2(R_i) \,, \tag{2.5}$$

where $l(R_i)$ is the Dynkin index of R_i, and $C_2(G)$, $C_2(R_i)$ are respectively the quadratic Casimir invariant of the adjoint representation of G, and of the R_i representation. A natural question to ask is what happens at higher loop orders. A very interesting result is that the conditions (2.5) are necessary and sufficient for finiteness at the two-loop level [19, 20].

The one- and two-loop finiteness conditions (2.5) restrict considerably the possible choices of the irreps. R_i for a given group G as well as the Yukawa couplings in the superpotential (2.4) [21]–[23]. Note in particular that the finiteness conditions cannot be applied to the supersymmetric standard model (SSM), since the presence of a U(1) gauge group is incompatible with the first of the conditions (2.5), due to $C_2[\mathrm{U}(1)] = 0$. This leads to the expectation that finiteness should be attained at the grand unified level only, the SSM being just the corresponding, low-energy, effective theory.

The finiteness conditions impose relations between gauge and Yukawa couplings. Therefore, we have to guarantee that such relations leading to a reduction of the couplings hold at any renormalization point. The necessary, but also sufficient, condition for this to happen is to require that such relations are solutions to the reduction equations (REs) to

JHEP12(2004)026

all orders. Specifically there exists a very interesting theorem [12] which guarantees the vanishing of the β-functions to all orders in perturbation theory, if we demand reduction of couplings, and that all the one-loop anomalous dimensions of the matter field in the completely and uniquely reduced theory vanish identically.

3. Soft supersymmetry breaking in $N = 1$ FUTS

The above described method of reducing the dimensionless couplings has been extended [24] to the soft supersymmetry breaking (SSB) dimensionful parameters of $N = 1$ supersymmetric theories. In addition it was found [25] that RGI SSB scalar masses in general Gauge-Yukawa unified models satisfy a universal sum rule at one-loop, which was subsequently extended first up to two-loops [9] and then to all-loops [26].

To be more specific, consider the superpotential given by (2.4) along with the lagrangian for SSB terms

$$-\mathcal{L}_{\rm SB} = \frac{1}{6}\, h^{ijk}\, \phi_i\phi_j\phi_k + \frac{1}{2}\, b^{ij}\, \phi_i\phi_j + \frac{1}{2}\,(m^2)^j_i\, \phi^{*\,i}\phi_j + \frac{1}{2}\, M\,\lambda\lambda + \text{h.c.}\,, \tag{3.1}$$

where the ϕ_i are the scalar parts of the chiral superfields Φ_i , λ are the gauginos and M their unified mass. Since we would like to consider only finite theories here, we assume that the one-loop β-function of the gauge coupling g vanishes. We also assume that the reduction equations admit power series solutions of the form $C^{ijk} = g\,\sum_{n=0}\rho^{ijk}_{(n)} g^{2n}$. According to the finiteness theorem of ref. [12], the theory is then finite to all orders in perturbation theory, if, among others, the one-loop anomalous dimensions $\gamma_i^{j(1)}$ vanish. The one- and two-loop finiteness for h^{ijk} can be achieved [19, 27] by imposing the condition

$$h^{ijk} = -MC^{ijk} + \cdots = -M\rho^{ijk}_{(0)}\, g + O(g^5)\,. \tag{3.2}$$

In addition, it was found [9] that one and two-loop finiteness requires that the following two-loop sum rule for the soft scalar masses has to be satisfied

$$\frac{(\;m_i^2 + m_j^2 + m_k^2\;)}{MM^\dagger} = 1 + \frac{g^2}{16\pi^2}\,\Delta^{(2)} + O(g^4)\,, \tag{3.3}$$

where $\Delta^{(2)}$ is the two-loop correction,

$$\Delta^{(2)} = -2\sum_l \left[\left(\frac{m_l^2}{MM^\dagger}\right) - \left(\frac{1}{3}\right)\right]\; T(R_l)\,, \tag{3.4}$$

which vanishes for the universal choice [27]. Furthermore, it was found [28] that the relation

$$h^{ijk} = -M(C^{ijk})' \equiv -M\frac{dC^{ijk}(g)}{d\ln g}\,, \tag{3.5}$$

among couplings is all-loop RGI. Moreover, the progress made using the spurion technique leads to all-loop relations among SSB β-functions [10, 28] and [30]–[32], which allowed to find the all-loop RGI sum rule [26] in the Novikov-Shifman-Vainstein-Zakharov scheme [33].

4. Search for realistic FUTs based on product gauge groups

Let us now examine the possibility of constructing realistic FUTs based on product gauge groups. Consider the gauge group $\mathrm{SU}(N)_1 \times \mathrm{SU}(N)_2 \times \cdots \times \mathrm{SU}(N)_k$ with n_f copies of the supersymmetric multiplet $(N, N^*, 1, \ldots, 1) + (1, N, N^*, \ldots, 1) + \cdots + (N^*, 1, 1, \ldots, N)$. The one-loop β-function coefficient in the renormalization-group equation of each $\mathrm{SU}(N)$ gauge coupling is simply given by

$$b = \left(-\frac{11}{3} + \frac{2}{3}\right) N + n_f \left(\frac{2}{3} + \frac{1}{3}\right) \left(\frac{1}{2}\right) 2N = -3N + n_f N\,. \tag{4.1}$$

This means that $n_f = 3$ is a solution of the equation $b = 0$, independently of the values of N and k. Since $b = 0$ is a necessary condition for a finite field theory, the existence of three families of quarks and leptons is natural in such models. (This is true of course only if the matter content is exactly as given above. Other $\mathrm{SU}(N)^k$ models exist with very different, and rather *ad hoc*, supermultiplet structure. They are not included in our discussion.)

Next let us examine if this class of models can meet the obvious requirements in every unified theory, namely (i) that it leads to the SM or the MSSM at low energies, and (ii) that it predicts correctly $sin^2\theta_W$.

Let $N = 3$ and $k = 3$, then we have the well-known example of $\mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ [34, 35], with quarks transforming as

$$q = \begin{pmatrix} d & u & h \\ d & u & h \\ d & u & h \end{pmatrix} \sim (3, 3^*, 1)\,, \qquad q^c = \begin{pmatrix} d^c & d^c & d^c \\ u^c & u^c & u^c \\ h^c & h^c & h^c \end{pmatrix} \sim (3^*, 1, 3)\,, \tag{4.2}$$

and leptons transforming as

$$\lambda = \begin{pmatrix} N & E^c & \nu \\ E & N^c & e \\ \nu^c & e^c & S \end{pmatrix} \sim (1, 3, 3^*)\,. \tag{4.3}$$

If we switch the first and third rows of q^c together with the first and third columns of λ, we obtain the alternative left-right model first proposed in ref. [36] in the context of superstring-inspired E_6. The breaking down of $\mathrm{SU}(3)^3$ to $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{Y_L+Y_R}$ is achieved with the (3,3) entry of λ, and the further breaking of $\mathrm{SU}(2)_R \times \mathrm{U}(1)_{Y_L+Y_R}$ to $\mathrm{U}(1)_Y$ with the (3,1) entry.

Let $N = 3$ and $k = 4$, then one example is the extension to include the chiral color of ref. [37]. Here $\mathrm{SU}(3)_C$ is split up into $\mathrm{SU}(3)_{CL}$ and $\mathrm{SU}(3)_{CR}$. This implies the existence of a neutral supermultiplet η transforming as (N^*, N) under these two groups. Let $\langle\eta_{11}\rangle = \langle\eta_{22}\rangle = \langle\eta_{33}\rangle$, then $\mathrm{SU}(3)_{CL} \times \mathrm{SU}(3)_{CR}$ breaks back down to $\mathrm{SU}(3)_C$ as desired. However at this scale,

$$\alpha_s^{-1} = \alpha_{sL}^{-1} + \alpha_{sR}^{-1} \tag{4.4}$$

and since α_{sL} and α_{sR} are to be unified with α_L and α_R, the predicted value of α_s would be too small. Thus this is not a candidate model of unification, unless the particle content is also extended [38], in which case finiteness would be lost.

JHEP12(2004)026

Another possibility to consider is the quartification model of ref. [39]. Here unification is possible but only in the nonsupersymmetric case. In fact, $\sin^2\theta_W = 1/3$ instead of the canonical $3/8$, and the unification scale of this model is only 4×10^{11} GeV.

Let us now turn to the interesting $N=4$ and $k=3$ case [15]. The obvious choice is $SU(4)_C \times SU(4)_L \times SU(4)_R$, where $SU(4)_C$ is the Pati-Salam color gauge group [40]. In that case, the quarks and leptons should transform as

$$f = \begin{pmatrix} d & u & y & x \\ d & u & y & x \\ d & u & y & x \\ e & \nu & a & v \end{pmatrix} \sim (4, 4^*, 1)\,, \qquad f^c = \begin{pmatrix} d^c & d^c & d^c & e^c \\ u^c & u^c & u^c & \nu^c \\ y^c & y^c & y^c & a^c \\ x^c & x^c & x^c & v^c \end{pmatrix} \sim (4^*, 1, 4)\,. \tag{4.5}$$

We see immediately that there have to be new heavy particles, i.e. the x and y quarks and the v and a leptons. In addition, we need to consider the $h \sim (1, 4, 4^*)$ supermultiplet.

The unification of quarks and leptons within $SU(4)_C$ implies that their electric charge Q should be given by

$$Q = \frac{1}{2}(B-L) + I_{3L} + I_{3R}\,. \tag{4.6}$$

However, the electric charges of the new heavy particles are not yet fixed. This is because $SU(4)$ contains two disjoint $SU(2)$ subgroups, one of which may be the usual $SU(2)_L$ or $SU(2)_R$, but the other is new. Therefore, another valid formula for Q is given by

$$Q = \frac{1}{2}(B-L) + I_{3L} + I_{3R} + I'_{3L} + I'_{3R}\,. \tag{4.7}$$

The quarks and leptons do not transform under $SU(2)'_L$ or $SU(2)'_R$, so their electric charges are not affected.

Using eq. (4.6), the charges of f, f^c, and h are respectively

$$Q_f = \begin{pmatrix} -1/3 & 2/3 & 1/6 & 1/6 \\ -1/3 & 2/3 & 1/6 & 1/6 \\ -1/3 & 2/3 & 1/6 & 1/6 \\ -1 & 0 & -1/2 & -1/2 \end{pmatrix}, \tag{4.8}$$

$$Q_{f^c} = \begin{pmatrix} 1/3 & 1/3 & 1/3 & 1 \\ -2/3 & -2/3 & -2/3 & 0 \\ -1/6 & -1/6 & -1/6 & 1/2 \\ -1/6 & -1/6 & -1/6 & 1/2 \end{pmatrix}, \tag{4.9}$$

$$Q_h = \begin{pmatrix} 0 & 1 & 1/2 & 1/2 \\ -1 & 0 & -1/2 & -1/2 \\ -1/2 & 1/2 & 0 & 0 \\ -1/2 & 1/2 & 0 & 0 \end{pmatrix}. \tag{4.10}$$

Using eq. (4.7), they are instead

$$Q_f = \begin{pmatrix} -1/3 & 2/3 & -1/3 & 2/3 \\ -1/3 & 2/3 & -1/3 & 2/3 \\ -1/3 & 2/3 & -1/3 & 2/3 \\ -1 & 0 & -1 & 0 \end{pmatrix}, \tag{4.11}$$

$$Q_{f^c} = \begin{pmatrix} 1/3 & 1/3 & 1/3 & -1 \\ -2/3 & -2/3 & -2/3 & 0 \\ 1/3 & 1/3 & 1/3 & -1 \\ -2/3 & -2/3 & -2/3 & 0 \end{pmatrix}, \tag{4.12}$$

$$Q_h = \begin{pmatrix} 0 & 1 & 0 & 1 \\ -1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & -1 & 0 \end{pmatrix}. \tag{4.13}$$

The two different charge assignments result in two different values of

$$\sin^2\theta_W = \frac{\sum I_{3L}^2}{\sum Q^2} \tag{4.14}$$

at the unification scale. Whereas it is equal to 3/8 as usual in the former, it becomes 3/14 in the latter, which is not realistic. Therefore we will discuss further only the case with the charge assignments of eqs. (4.8–4.10).

Since we do not admit any other matter supermultiplets, the symmetry breaking of $\mathrm{SU}(4)_C \times \mathrm{SU}(4)_L \times \mathrm{SU}(4)_R$ must be achieved with the vacuum expectation values of the neutral scalar components of f, f^c, and h. The best we can do is to let all the (3,3), (3,4), (4,3), and (4,4) entries of h acquire vacuum expectation values, but then the $\mathrm{SU}(4)^3$ symmetry is only broken down to $\mathrm{SU}(4)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{L+R}$. The extra unwanted U(1) is necessarily present because in the decomposition of $\mathrm{SU}(4)_L$ and $\mathrm{SU}(4)_R$ to their $\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ subgroups, the diagonal subgroup $\mathrm{U}(1)_{L+R}$ cannot be broken by the representation $(1,4,4^*)$. This problem persists even after the breaking of $\mathrm{SU}(4)_C \times \mathrm{SU}(2)_R$ by the (2,4) entry of f^c to $\mathrm{SU}(3)_C \times \mathrm{U}(1)_Y$.

Since the unbroken U(1) couples to all particles, including the known quarks and leptons, this model cannot be viable phenomenologically. We are thus forced to conclude that $\mathrm{SU}(4)_C \times \mathrm{SU}(4)_L \times \mathrm{SU}(4)_R$ with only the matter content of f, f^c, and h is not a suitable candidate for a finite theory of all particles.

There is another important constraint for a realistic $\mathrm{SU}(N)^k$ theory of quarks and leptons, i.e. the proper masses must be obtained. Excluding naturally nonrenormalizable terms in the superpotential, then only bilinear and trilinear terms are allowed. For the matter content assumed here, it would be zero unless $N = 3$ or $k = 3$. (We exclude $N = 2$ or $k = 2$ for obvious reasons.) If $N = 3$, then we have an invariant from the product of three $(3, 3^*)$ supermultiplets. If $k = 3$, then the invariant $(N, N^*, 1)(1, N, N^*)(N^*, 1, N)$ may be formed. Therefore, this discussion leads us naturally to the case $\mathrm{SU}(3)^3$.

5. An all-loop $\mathrm{SU}(3)^3$ FUT

Here we will discuss in some detail the supersymmetric $\mathrm{SU}(3)^3$ FUT with three families. In general a supersymmetric E_6 model in four dimensions is easily obtained in compactifications of a ten-dimensional E_8, appearing in the heterotic string, over Calabi-Yau spaces [41]. Even more interesting is the possibility to obtain softly broken supersymmetric E_6 type models via coset space dimensional reduction [42, 43] in compactifications using non-symmetric coset spaces [44]. Subsequently the $\mathrm{SU}(3)^3$ can emerge using the Wilson fluxes [41, 45] in a straightforward way. What is less obvious to obtain is the spontaneous symmetry breaking of $\mathrm{SU}(3)^3$ down to the MSSM, however it has been done already some time ago [46]. It requires introducing eight superfield of the type (λ, q, q^c) and five corresponding mirror superfields $(\bar{\lambda}, \bar{q}, \bar{q^c})$. The details of this construction are given in ref. [46]. Therefore what remains as an open question is how to obtain the complete and detailed chain of breakings of the ten-dimensional E_8 down to the four-dimensional MSSM, but this is deeply related to the most fundamental problem of string theory, and will not be addressed further here. For our purposes, following [46], we consider a supersymmetric $\mathrm{SU}(3)^3$ model with three families holding between the Planck M_P and the unification M_{GUT} scales, which breaks spontaneously down to the MSSM at M_{GUT}.

In order for all the gauge couplings to be equal at M_{GUT}, as is suggested by the LEP results [47], the cyclic symmetry Z_3 must be imposed, i.e.

$$q \to \lambda \to q^c \to q \,, \tag{5.1}$$

where q and q^c are given in eq. (4.2) and λ in eq. (4.3). Then, according to the discussion in section 3, the first of the finiteness conditions (2.5) for one-loop finiteness, namely the vanishing of the gauge β-function is satisfied.

Next let us consider the second condition, i.e. the vanishing of the anomalous dimensions of all superfields. To do that first we have to write down the superpotential. If there is just one family, then there are only two trilinear invariants, which can be constructed respecting the symmetries of the theory, and therefore can be used in the superpotential as follows

$$f\ Tr(\lambda q^c q) + \frac{1}{6} f'\ \epsilon_{ijk}\epsilon_{abc}(\lambda_{ia}\lambda_{jb}\lambda_{kc} + q^c_{ia}q^c_{jb}q^c_{kc} + q_{ia}q_{jb}q_{kc}) \,. \tag{5.2}$$

In this case, the condition for vanishing anomalous dimension of each superfield is given by [12, 13, 8, 9, 10]

$$\frac{1}{2}(3|f|^2 + 2|f'|^2) = 2\left(\frac{4}{3}g^2\right) . \tag{5.3}$$

Quark and leptons obtain masses when the scalar parts of the superfields $(\tilde{N}, \tilde{N}^c)$ obtain vacuum expectation values (vevs),

$$m_d = f\langle\tilde{N}\rangle, \quad m_u = f\langle\tilde{N}^c\rangle \,, \qquad m_e = f'\langle\tilde{N}\rangle, \quad m_\nu = f'\langle\tilde{N}^c\rangle \,. \tag{5.4}$$

With three families, the most general superpotential contains 11 f couplings, and 10 f' couplings, subject to 9 conditions, due to the vanishing of the anomalous dimensions of each superfield. The conditions are the following

$$\sum_{j,k} f_{ijk}(f_{ljk})^* + \frac{2}{3}\sum_{j,k} f'_{ijk}(f'_{ljk})^* = \frac{16}{9}g^2\delta_{il} \,, \tag{5.5}$$

where

$$f_{ijk} = f_{jki} = f_{kij} \,, \tag{5.6}$$

$$f'_{ijk} = f'_{jki} = f'_{kij} = f'_{ikj} = f'_{kji} = f'_{jik} \,. \tag{5.7}$$

JHEP12(2004)026

Quarks and leptons receive masses when the scalar part of the superfields $\tilde{N}_{1,2,3}$ and $\tilde{N}^c_{1,2,3}$ obtain vevs as follows

$$(\mathcal{M}_d)_{ij} = \sum_k f_{kij} \langle \tilde{N}_k \rangle \,, \qquad (\mathcal{M}_u)_{ij} = \sum_k f_{kij} \langle \tilde{N}^c_k \rangle \,, \tag{5.8}$$

$$(\mathcal{M}_e)_{ij} = \sum_k f'_{kij} \langle \tilde{N}_k \rangle \,, \qquad (\mathcal{M}_\nu)_{ij} = \sum_k f'_{kij} \langle \tilde{N}^c_k \rangle \,. \tag{5.9}$$

Since we want to have, among other conditions, gauge coupling unification, we will assume that the particle content of our finite SU(3)^3 model below M_U is that of the MSSM with three fermion families, but only two Higgs doublets. Therefore we have to choose the linear combinations $\tilde{N}^c = \sum_i a_i \tilde{N}^c_i$ and $\tilde{N} = \sum_i b_i \tilde{N}_i$ to play the role of the two Higgs doublets, which will be responsible for the electroweak symmetry breaking. This can be done by choosing appropriately the masses in the superpotential [23], since they are not constrained by the finiteness conditions. Moreover, we choose that the two Higgs doublets are predominately coupled to the third generation. Then these two Higgs doublets couple to the three families differently, thus providing the freedom to understand their different masses and mixings.

Assuming for our purposes here that all f' couplings vanish[1] an isolated solution eq. (5.5) is

$$f^2 = f^2_{111} = f^2_{222} = f^2_{333} = \frac{16}{9} g^2 \,. \tag{5.10}$$

Hence we start at M_{GUT} with different Yukawa couplings for all the quarks

$$f_t = f a_3 \,, \qquad f_c = f a_2 \,, \qquad f_u = f a_1 \,, \tag{5.11}$$

$$f_b = f b_3 \,, \qquad f_s = f b_2 \,, \qquad f_d = f b_1 \,, \tag{5.12}$$

which is similar to the MSSM except that f is fixed by finiteness at M_{GUT}, and $a_3 \simeq 1$, $b_3 \simeq 1$, by construction, and therefore we have that $f_t \simeq f_b \simeq f$ at M_{GUT}. As for the lepton masses, because all f' couplings have been fixed to be zero at this order, in principle they are expected to appear radiatively induced by the scalar lepton masses appearing in the SSB sector of the theory. Unfortunately though, due to the finiteness conditions (3.2) they cannot appear radiatively and remain as a problem for further study. On the other hand it should be stressed that we can certainly let f' be non-vanishing in eq. (5.5) and thus introduce lepton masses in the model. Then the real price to be paid is basically aesthetic since the model in turn becomes finite only up to two-loops since the corresponding solution of eq. (5.5) is not an isolated one any more. However, given that the analysis we do in the next section takes into account RGEs up to two-loops, there is no practical cost in introducing non-zero f'. We include this possibility in our analysis in section 6.

Although we present the results of a more complete analysis in the next section, we find instructive to describe here the situation concerning the top quark mass prediction at

[1] In supersymmetric theories this can always be done due to the non-renormalization theorem [48], which guarantees that these terms will not appear radiatively. In general this is not the case in the presence of supersymmetry breaking terms, however finiteness imposes tight conditions in this respect too.

JHEP12(2004)026

one-loop level ignoring the SSB sector. In this approximate analysis, we run the MSSM renormalization group equations at one-loop, using our boundary condition $f^2 = (16/9)g^2$ at the M_{GUT} scale as follows

$$8\pi^2 \left(\frac{dg_3^2}{dt}\right) = -3g_3^4\,, \tag{5.13}$$

$$8\pi^2 \left(\frac{dg_2^2}{dt}\right) = g_2^4\,, \tag{5.14}$$

$$8\pi^2 \left(\frac{dg_1^2}{dt}\right) = \frac{33}{5}g_1^4\,, \tag{5.15}$$

$$8\pi^2 \left(\frac{df_t^2}{dt}\right) = f_t^2\left(6f_t^2 + f_b^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\right), \tag{5.16}$$

$$8\pi^2 \left(\frac{df_b^2}{dt}\right) = f_b^2\left(6f_b^2 + f_t^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\right). \tag{5.17}$$

The g_i^2s are easily solved as functions of $t = \ln(M_{GUT}/M)$:

$$\alpha_3(M)^{-1} = \alpha_3(M_{\rm GUT})^{-1} - \left(\frac{3}{2\pi}\right)\ln\left(\frac{M_{\rm GUT}}{M}\right), \tag{5.18}$$

$$\alpha_2(M)^{-1} = \alpha_2(M_{\rm GUT})^{-1} + \left(\frac{1}{2\pi}\right)\ln\left(\frac{M_{\rm GUT}}{M}\right), \tag{5.19}$$

$$\alpha_1(M)^{-1} = \alpha_1(M_{\rm GUT})^{-1} + \left(\frac{33}{10\pi}\right)\ln\left(\frac{M_{\rm GUT}}{M}\right), \tag{5.20}$$

where $\alpha_i = g_i^2/4\pi$. Using the MSSM boundary conditions from the unification of the gauge couplings at one-loop and the constraints of the present model we have

$$\alpha_i(M_{\rm GUT}) = 0.0413\,, \tag{5.21}$$

$$\alpha_t(M_{\rm GUT}) = \alpha_b(M_{\rm GUT}) = \left(\frac{16}{9}\right)\alpha_i(M_{\rm GUT})\,. \tag{5.22}$$

Then we integrate the two differential equations (5.16) and (5.17), from $t = \ln(M_{\rm GUT}/M_{\rm EW})$ to $t = 0$, to determine f_t and f_b at the electroweak scale M_{EW}. Then $m_t = f_t v_u$ and $m_b = f_b v_d$, with v_u and V_d satisfying the condition $v_u^2 + v_d^2 = v^2$, $v = 174.3$ GeV. Thus given m_b, we can obtain m_t.

6. Predictions and conclusions

The gauge symmetry $\mathrm{SU}(3)^3$ is spontaneously broken down to the MSSM at $M_{\rm GUT}$, and the finiteness conditions do not restrict the renormalization properties at low energies. Therefore, below M_{GUT} all couplings and masses of the theory run according to the RGEs of the MSSM. The remnants of the all-loop FUT $\mathrm{SU}(3)^3$ are the boundary conditions on the gauge and Yukawa couplings (5.10), the $h = -MC$ relation, and the soft scalar-mass sum rule (3.3) at $M_{\rm GUT}$, which, when applied to the present model, takes the form

$$m_{H_u}^2 + m_{\tilde{t}^c}^2 + m_{\tilde{q}}^2 = M^2 \tag{6.1}$$

$$m_{H_d}^2 + m_{\tilde{b}^c}^2 + m_{\tilde{q}}^2 = M^2\,. \tag{6.2}$$

JHEP12(2004)026

Thus we examine the evolution of these parameters according to their RGEs up to two-loops for dimensionless parameters and at one-loop for dimensionful ones imposing the corresponding boundary conditions. We further assume a unique supersymmetry breaking scale M_s (defined as the average of the mass of the stops) and therefore below that scale the effective theory is just the SM.

We consider two versions of the model:

I) The all-loop finite one in which f' vanishes and eq. (5.10) holds.

II) A two-loop finite version, in which we keep f' non-vanishing in eq. (5.5), and we use it to introduce the lepton masses.

The predictions for the top quark mass m_t are $\sim 183\,\mathrm{GeV}$ for $\mu < 0$ in model I, whereas for model II it is $176 - -179\,\mathrm{GeV}$ for $\mu < 0$, and $170 - -173\,\mathrm{GeV}$ for $\mu > 0$. Recall that the bottom quark mass m_b is an input in FUT I and m_τ in FUT II.

Comparing these predictions with the most recent experimental value $m_t^{exp} = (178.0 \pm 4.3)\,\mathrm{GeV}$ [49], and recalling that the theoretical values for m_t may suffer from a correction of $\sim 4\%$ [10], we see that they are consistent with the experimental data.

In the SSB sector, besides the constraints imposed by finiteness we further require

1. successful radiative electroweak symmetry breaking, and
2. $m^2_{\tilde{\tau},\tilde{b},\tilde{t}} > 0$.

As an additional constraint, we take into account the $BR(b \to s\gamma)$ [50]. We do not take into account, though, constraints coming from the muon anomalous magnetic moment (g-2) in this work, which would exclude a small region of the parameter space.

Our numerical analysis shows the following results for the two models: In the case of FUT I it is possible to find regions of parameter space which comply with all the above requirements both for the case where we have universal boundary conditions ($m_i^2 = m_j^2 = m_k^2 = M^2/3$), and for the case where we apply the sum rule eq. (3.3). In the case of universal boundary conditions and $\mu < 0$, $m_t \sim 183$ GeV, the Higgs mass is $\sim 131 - 132$ GeV, $\tan\beta \sim 50 - 51$, and the spectrum is rather heavy, the allowed region of parameter space starting with an LSP which is a neutralino $m_{\chi^0} \sim 825$ GeV for a value of $M \sim 1800\,\mathrm{GeV}$. In the case the sum rule is applied we have one more free parameter, which is $m_{\tilde{q}^c} = m_{\tilde{q}}$ at the GUT scale. In this case we obtain a $\tan\beta \sim 47 - 54$, and the Higgs mass is $\sim 130 - 132$ GeV. The main difference between the universal boundary conditions and the sum rule comes in the sparticle spectrum, which can now start with an LSP at $m_{\chi^0} \sim 450\,\mathrm{GeV}$, for a boundary condition of $M \sim 1800\,\mathrm{GeV}$. In the case that $\mu > 0$ we do not find solutions which satisfy all the above requirements.

In the second version of the model FUT II, we have the following boundary conditions for the Yukawa couplings

$$f^2 = r\left(\frac{16}{9}\right)g^2\,, \tag{6.3}$$

$$f'^2 = (1-r)\left(\frac{8}{3}\right)g^2\,. \tag{6.4}$$

In this case, we do not have an all-loop finite model, since the solution is a parametric one, but it is the price we pay to give masses to the leptons. As for the boundary conditions of the soft scalars, we only have the universal case. This is because, applying the sum rule (3.3) to the superpotential with $f' \neq 0$ implies that $m_q^2 = m_{q^c}^2 = m_{H^{u,d}}^2 = M^2/3$, which is again the universal boundary condition. For the numerical analysis we fix the m_τ mass to obtain m_t and m_b. Taking $\mu < 0$, and for the experimentally allowed value of $m_b(m_b) = 4.1 - 4.4\,\mathrm{GeV}$ [51], the value of m_t goes from $\sim 176 - 179\,\mathrm{GeV}$. In this case $\tan\beta \sim 48 - 53$, and $m_H \sim 122 - 129\,\mathrm{GeV}$, with a charged LSP $m_{\tilde{\tau}} \sim 400 - 1000\,\mathrm{GeV}$, depending directly on the value of M, which varies from $\sim 1200 - 2200\,\mathrm{GeV}$ in this case.

Now for $\mu > 0$, the value of m_t compatible with the experimentally allowed value of m_b, goes from $\sim 170 - 173\,\mathrm{GeV}$, clearly the preferred value being the latter. For this range of values of m_t we obtain $\tan\beta \sim 58 - 62$, and $m_H \sim 120 - 125\,\mathrm{GeV}$, also with a charged LSP $m_{\tilde{\tau}} \sim 300 - 600\,\mathrm{GeV}$, again depending directly on the value of M, which varies from $\sim 1300 - 2000\,\mathrm{GeV}$.

We could go further and consider another version of the $\mathrm{SU}(3)^3$ model. For instance, if we impose global SU(3) as a family symmetry [16, 52], then there is only one Yukawa coupling in the superpotential, which leads to the following unique relation among Yukawa and gauge couplings

$$f^2 = \frac{8}{9} g^2 \,. \tag{6.5}$$

However both $\mathcal{M}_u$ and $\mathcal{M}_d$ in eq. (5.8) must now be antisymmetric in family space, resulting in one zero and two equal mass eigenvalues for each, which is not a realistic case. Note moreover, that the terms proportional to f' in the superpotential eq. (5.2) are not allowed to appear in the cases of refs. [15, 16] unless $N = 3$, and therefore they share the problem of the FUT I model, where we have chosen $f' = 0$.

Acknowledgments

We acknowledge useful discussions with J. Erler, and use of some of his results for the quark masses from his GAPP program [53]. We also acknowledge useful discussions with S. Heinemeyer. E.M. and G.Z. thank UNAM for its great hospitality during a recent very productive visit.

References

[1] A. Connes, M.R. Douglas and A. Schwarz, *Noncommutative geometry and matrix theory: compactification on tori*, *J. High Energy Phys.* **02** (1998) 003 [hep-th/9711162].

[2] N. Seiberg and E. Witten, *String theory and noncommutative geometry*, *J. High Energy Phys.* **09** (1999) 032 [hep-th/9908142].

[3] I. Jack and D.R.T. Jones, *Ultra-violet finiteness in noncommutative supersymmetric theories*, *New J. Phys.* **3** (2001) 19 [hep-th/0109195].

[4] S. Mandelstam, *Light cone superspace and the finiteness of the $N = 4$ model*, CERN-TH-3385.

[5] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[6] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109].

[7] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[8] D. Kapetanakis, M. Mondragón and G. Zoupanos, *Finite unified models*, *Zeit. f. Phys.* **C60** (1993) 181;
M. Mondragón and G. Zoupanos, *Finite unified theories and the top quark mass*, *Nucl. Phys.* **37C** *(Proc. Suppl.)* (1995) 98.

[9] T. Kobayashi, J. Kubo, M. Mondragón and G. Zoupanos, *Constraints on finite soft supersymmetry-breaking terms*, *Nucl. Phys.* **B 511** (1998) 45 [hep-ph/9707425].

[10] For an extended discussion and a complete list of references see: M. Mondragon and G. Zoupanos, *Higgs mass prediction in finite unified theories*, *Acta Phys. Polon.* **34** (2003) 5459;
T. Kobayashi, J. Kubo, M. Mondragón, and G. Zoupanos, *Finite unification*, *Surveys High Energy Phys* **16** (2001) 87.

[11] W. Zimmermann, *Reduction in the number of coupling parameters*, *Commun. Math. Phys.* **97** (1985) 211;
R. Oehme and W. Zimmermann, *Relation between effective couplings for asymptotically free models*, *Commun. Math. Phys.* **97** (1985) 569;
See also N.P. Chang, *Eigenvalue conditions and asymptotic freedom for Higgs scalar gauge theories*, *Phys. Rev.* **D 10** (1974) 2706;
E. Ma, *Modified quantum chromodynamics, 1. Exact global color symmetry and asymptotic freedom*, *Phys. Rev.* **D 17** (1978) 623; *Fixing the Higgs boson mass*, *Phys. Rev.* **D 31** (1985) 1143;
S. Nandi and W.C. Ng, *Can coupling constants be related?*, *Phys. Rev.* **D 20** (1979) 972.

[12] C. Lucchesi, O. Piguet and K. Sibold, *Vanishing beta functions in $N = 1$ supersymmetric gauge theories*, *Helv. Phys. Acta* **61** (1988) 321;
C. Lucchesi and G. Zoupanos, *All order finiteness in $N = 1$ SYM theories: criteria and applications*, *Fortschr. Phys.* **45** (1997) 129.

[13] A.Z. Ermushev, D.I. Kazakov and O.V. Tarasov, *Finite $N = 1$ supersymmetric Grand Unified Theories*, *Nucl. Phys.* **B 281** (1987) 72;
D.I. Kazakov, *Finite $N = 1$ SUSY gauge field theories*, *Mod. Phys. Lett.* **A 2** (1987) 663.

[14] R.G. Leigh and M.J. Strassler, *Exactly marginal operators and duality in four-dimensional $N = 1$ supersymmetric gauge theory*, *Nucl. Phys.* **B 447** (1995) 95;
P.C. Argyres, K.A. Intriligator, R.G. Leigh and M.J. Strassler, *On inherited duality in $N = 1$ $D = 4$ supersymmetric gauge theories*, *J. High Energy Phys.* **04** (2000) 029 [hep-th/9910250].

[15] L.E. Ibáñez, *A chiral $D = 4$, $N = 1$ string vacuum with a finite low energy effective field theory*, *J. High Energy Phys.* **07** (1998) 002 [hep-th/9802103].

[16] S. Kachru and E. Silverstein, *4d conformal theories and strings on orbifolds*, *Phys. Rev. Lett.* **80** (1998) 4855 [hep-th/9802183].

JHEP12(2004)026

[17] A. Brignole, L.E. Ibáñez, C. Muñoz and C. Scheich, *Some issues in soft SUSY breaking terms from dilaton/moduli sectors*, *Z. Physik* **C 74** (1997) 157 [hep-ph/9508258].

[18] P.G. Camara, L.E. Ibáñez and A.M. Uranga, *Flux-induced SUSY-breaking soft terms*, *Nucl. Phys.* **B 689** (2004) 195 [hep-th/0311241].

[19] D.R.T. Jones, L. Mezincescu and Y.P. Yao, *Soft breaking of two loop finite $N = 1$ supersymmetric gauge theories*, *Phys. Lett.* **B 148** (1984) 317.

[20] A. Parkes and P.C. West, *Finiteness in rigid supersymmetric theories*, *Phys. Lett.* **B 138** (1984) 99;
D.R.T. Jones and L. Mezincescu, *The beta function in supersymmetric Yang-Mills theory*, *Phys. Lett.* **B 136** (1984) 242.

[21] S. Hamidi, J. Patera and J.H. Schwarz, *Chiral two loop finite supersymmetric theories*, *Phys. Lett.* **B 141** (1984) 349;
X.D. Jiang and X.J. Zhou, *Finite $N = 1$ supersymmetric theories of* $\mathrm{SU}(N)$, *Phys. Lett.* **B 197** (1987) 156; *Finite $N = 1$ supersymmetric theories of classical groups*, *Phys. Lett.* **B 216** (1989) 160.

[22] S. Hamidi and J.H. Schwarz, *A realistic finite unified theory?*, *Phys. Lett.* **B 147** (1984) 301;
D.R.T. Jones and S. Raby, *A two loop finite supersymmetric* $\mathrm{SU}(5)$ *theory: towards a theory of fermion masses*, *Phys. Lett.* **B 143** (1984) 137;
J.E. Bjorkman, D.R.T. Jones and S. Raby, *Low-energy predictions of a two loop finite supersymmetric* $\mathrm{SU}(5)$ *theory*, *Nucl. Phys.* **B 259** (1985) 503.

[23] J. León, J. Pérez-Mercader, M. Quirós and J. Ramírez-Mittelbrunn, *A sensible finite* $\mathrm{SU}(5)$ *SUSY GUT?*, *Phys. Lett.* **B 156** (1985) 66.

[24] J. Kubo, M. Mondragón and G. Zoupanos, *Perturbative unification of soft supersymmetry-breaking terms*, *Phys. Lett.* **B 389** (1996) 523 [hep-ph/9609218].

[25] Y. Kawamura, T. Kobayashi and J. Kubo, *Soft scalar-mass sum rule in gauge-Yukawa unified models and its superstring interpretation*, *Phys. Lett.* **B 405** (1997) 64 [hep-ph/9703320].

[26] T. Kobayashi, J. Kubo and G. Zoupanos, *Further all-loop results in softly-broken supersymmetric gauge theories*, *Phys. Lett.* **B 427** (1998) 291 [hep-ph/9802267].

[27] I. Jack and D.R.T. Jones, *Soft supersymmetry breaking and finiteness*, *Phys. Lett.* **B 333** (1994) 372 [hep-ph/9405233].

[28] I. Jack, D.R.T. Jones and A. Pickering, *Renormalisation invariance and the soft beta functions*, *Phys. Lett.* **B 426** (1998) 73 [hep-ph/9712542].

[29] I. Jack and D.R.T. Jones, *Renormalization group invariance and universal soft supersymmetry breaking*, *Phys. Lett.* **B 349** (1995) 294 [hep-ph/9501395].

[30] J. Hisano and M.A. Shifman, *Exact results for soft supersymmetry breaking parameters in supersymmetric gauge theories*, *Phys. Rev.* **D 56** (1997) 5475 [hep-ph/9705417].

[31] Y. Yamada, *Two loop renormalization group equations for soft SUSY breaking scalar interactions: supergraph method*, *Phys. Rev.* **D 50** (1994) 3537 [hep-ph/9401241].

[32] D.I. Kazakov, *Finiteness of soft terms in finite $N = 1$ SUSY gauge theories* *Phys. Lett.* **B 412** (1998) 211.

JHEP12(2004)026

[33] V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *Exact Gell-Mann–Low function of supersymmetric Yang-Mills theories from instanton calculus*, *Nucl. Phys.* **B 229** (1983) 381; *Beta function in supersymmetric gauge theories: instantons versus traditional approach*, *Phys. Lett.* **B 166** (1986) 329;
M. Shifman, *Little miracles of supersymmetric evolution of gauge couplings*, *Int. J. Mod. Phys.* **A 11** (1996) 5761 and references therein.

[34] A. De Rújula, H. Georgi, and S. L. Glashow, in *Fifth Workshop on Grand Unification*, K. Kang, H. Fried, and P. Frampton eds., World Scientific, Singapore, 1984, p. 88;
K.S. Babu, X.-G. He and S. Pakvasa, *Neutrino masses and proton decay modes in* $\mathrm{SU}(3) \times \mathrm{SU}(3) \times \mathrm{SU}(3)$ *trinification*, *Phys. Rev.* **D 33** (1986) 763.

[35] G. Lazarides, C. Panagiotakopoulos and Q. Shafi, *Supersymmetric unification without proton decay*, *Phys. Lett.* **B 315** (1993) 325 [hep-ph/9306332].

[36] E. Ma, *Particle dichotomy and left-right decomposition of* E_6 *superstring models*, *Phys. Rev.* **D 36** (1987) 274.

[37] P.H. Frampton and S.L. Glashow, *Chiral color: an alternative to the standard model*, *Phys. Lett.* **B 190** (1987) 157;
see also G. Zoupanos, *Fermion mass generation and electroweak symmetry breaking from color forces*, *Phys. Lett.* **B 129** (1983) 315.

[38] A. Pérez-Lorenzana and W.A. Ponce, *An* $[\mathrm{SU}(3)]^4$ *supersymmetric Grand Unified model*, *Phys. Lett.* **B 464** (1999) 77 [hep-ph/9812402].

[39] K.S. Babu, E. Ma and S. Willenbrock, *Quark lepton quartification*, *Phys. Rev.* **D 69** (2004) 051301 [hep-ph/0307380].

[40] J.C. Pati and A. Salam, *Lepton number as the fourth color*, *Phys. Rev.* **D 10** (1974) 275.

[41] M.B. Green, J.H. Schwarz and E. Witten, *Superstring theory*, Cambridge University Press ,1987;
D. Lüst and S. Theisen, *Lectures on string theory*, *Lecture Physics*, Springer Verlag, Heidelberg 1989, vol. 346.

[42] P. Forgacs and N.S. Manton, *Space-time symmetries in gauge theories*, *Commun. Math. Phys.* **72** (1980) 15.

[43] D. Kapetanakis and G. Zoupanos, *Coset space dimensional reduction of gauge theories*, *Phys. Rept.* **219** (1992) 1;
Y.A. Kubyshin, I.P. Volobuev, J.M. Mourao and G. Rudolph, *Lecture notes in physics*, Springer Verlag, Heidelberg 1989, vol. 349;
F.A. Bais, K.J. Barnes, P. Forgacs and G. Zoupanos, *Dimensional reduction of gauge theories yielding unified models spontaneously broken to* $\mathrm{SU}(3) \times \mathrm{U}(1)$, *Nucl. Phys.* **B 263** (1986) 557.

[44] P. Manousselis and G. Zoupanos, *Dimensional reduction over coset spaces and supersymmetry breaking*, *J. High Energy Phys.* **03** (2002) 002 [hep-ph/0111125]; *Dimensional reduction of ten-dimensional supersymmetric gauge theories in the* $N = 1$, $D = 4$ *superfield formalism*, *J. High Energy Phys.* **11** (2004) 025 [hep-ph/0406207].

[45] Y. Hosotani, *Dynamical mass generation by compact extra dimensions*, *Phys. Lett.* **B 126** (1983) 309; *Dynamical gauge symmetry breaking as the Casimir effect*, *Phys. Lett.* **B 129** (1983) 193.

JHEP12(2004)026

[46] G. Lazarides and C. Panagiotakopoulos, *MSSM from SUSY trinification*, *Phys. Lett.* **B 336** (1994) 190 [hep-ph/9403317].

[47] U. Amaldi, W. de Boer and H. Furstenau, *Comparison of Grand Unified theories with electroweak and strong coupling constants measured at LEP*, *Phys. Lett.* **B 260** (1991) 447.

[48] J. Wess and J. Bagger, *Supersymmetry and supergravity*, Princeton University Press, 1983, ISBN 0-691-0832f0-7.

[49] CDF Collaborattion collaboration, P. Azzi et al., *Combination of CDF and D0 results on the top-quark mass*, hep-ex/0404010.

[50] P. Gambino and M. Misiak, *Quark mass effects in $\bar{B} \to X_s\gamma$*, *Nucl. Phys.* **B 611** (2001) 338 [hep-ph/0104034].

[51] Particle Data Group collaboration, S. Eidelman et al., *Review of particle physics*, *Phys. Lett.* **B 592** (2004) 1.

[52] J. Kubo, private communication.

[53] J. Erler, *Global fits to electroweak data using GAPP*, hep-ph/0005084.